JHEP HYPER VERSION

Received: *December 7, 1999*, Accepted: *January 24, 2000*

Four-neutrino mixing and long-baseline experiments

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Abstract: We consider the two four-neutrino schemes that are compatible with all neutrino oscillation data. We present the range of the corresponding mixing parameters allowed by the results of neutrino oscillation experiments. We discuss the implications for long-baseline experiments and we suggest the possibility to reveal the presence of CP-violating phases through a comparison of the oscillation probability measured in long-baseline experiments with the corresponding average oscillation probability measured in short-baseline experiments.

Keywords: Solar and Atmospheric Neutrinos, Neutrino Physics.

JHEP01(2000)032

Contents

1. Introduction

The three indications in favor of neutrino oscillations found in solar neutrino experiments [1], in atmospheric neutrino experiments [2] and in the accelerator LSND experiment [3] require the existence of three independent scales of neutrino mass-squared differences (see [4]): $\Delta m^2_{\text{sun}} \sim 10^{-6} - 10^{-4}\,\text{eV}^2$ (MSW) or $\Delta m^2_{\text{sun}} \sim 10^{-11} - 10^{-10}\,\text{eV}^2$ (VO) [5], $\Delta m^2_{\text{atm}} \sim 10^{-3} - 10^{-2}\,\text{eV}^2$ [6, 7], $\Delta m^2_{\text{SBL}} \sim 1\,\text{eV}^2$ [3]. Hence, at least four massive neutrinos must exist in nature.

Considering the minimal possibility of four massive neutrinos [8]–[15], we have the mixing relation

$$\nu_{\alpha L} = \sum_{k=1}^{4} U_{\alpha k}\, \nu_{kL}\,, \qquad (\alpha = e, \mu, \tau, s)\,, \tag{1.1}$$

that connects the flavor neutrino fields $\nu_{\alpha L}$ $(\alpha = e, \mu, \tau, s)$ with the fields ν_{kL} $(k = 1, \ldots, 4)$ of neutrinos with masses m_k. U is the $4{\times}4$ unitary mixing matrix. The field ν_{sL} describes sterile neutrinos, that do not participate to weak interactions and do not contribute to the invisible decay width of the Z-boson, whose measurement have shown that the number of light active neutrino flavors is three (see [16]), corresponding to ν_e, ν_μ and ν_τ.

The purpose of this paper is to present the implications of the results of neutrino oscillation experiments for the mixing of four light massive neutrinos. We will discuss also some implications for long-baseline neutrino oscillation experiments [17].

The plan of the paper is as follows. In section 2 we introduce the two four-neutrino mixing schemes that are allowed by the data and we review some of their properties. In section 3 we derive the constraints on four-neutrino mixing obtained from the results of neutrino oscillation experiments. In sections 4 and 5 we discuss some implications for long-baseline disappearance and appearance experiments, respectively. In section 6 we summarize our achievements.

In the discussion of long-baseline experiments we will neglect matter effects, that may be relevant for long-baseline accelerator experiments whose beams propagate for hundreds of kilometers in the crust of the Earth, but are rather complicated to study in the framework of four-neutrino mixing (see [15]) and will be treated in detail in another article [18].

2. The mixing schemes

It has been proved in [10, 11, 12] that among all the possible four-neutrino mixing schemes only two can accommodate the results of all neutrino oscillation experiments:

$$\text{(A)} \quad \underbrace{\overbrace{m_1 < m_2}^{\text{atm}} < \overbrace{m_3 < m_4}^{\text{sun}}}_{\text{SBL}}, \qquad \text{(B)} \quad \underbrace{\overbrace{m_1 < m_2}^{\text{sun}} < \overbrace{m_3 < m_4}^{\text{atm}}}_{\text{SBL}}. \tag{2.1}$$

These two spectra are characterized by the presence of two pairs of close masses separated by a gap of about 1 eV which provides the mass-squared difference[1] $\Delta m^2_{\text{SBL}} = \Delta m^2_{41}$ responsible for the short-baseline (SBL) oscillations observed in the LSND experiment. In the scheme A, we have $\Delta m^2_{\text{atm}} = \Delta m^2_{21}$ and $\Delta m^2_{\text{sun}} = \Delta m^2_{43}$, whereas in scheme B, $\Delta m^2_{\text{atm}} = \Delta m^2_{43}$ and $\Delta m^2_{\text{sun}} = \Delta m^2_{21}$. In order to treat simultaneously the two schemes A and B, let us rename the mass eigenstates indices as

$$c = 1\,, \quad d = 2\,, \quad a = 3\,, \quad b = 4\,, \qquad \text{in scheme A}\,, \tag{2.2}$$

$$a = 1\,, \quad b = 2\,, \quad c = 3\,, \quad d = 4\,, \qquad \text{in scheme B}\,. \tag{2.3}$$

With this convention, in both schemes ν_a and ν_b are the two massive neutrinos whose mass-squared difference is responsible for solar neutrino oscillations and ν_c and ν_d are the two massive neutrinos whose mass-squared difference generates atmospheric neutrino oscillations:

$$\Delta m^2_{\text{sun}} = \Delta m^2_{ba}\,, \qquad \Delta m^2_{\text{atm}} = \Delta m^2_{dc}\,. \tag{2.4}$$

For Δm^2_{SBL} in the following we will use the range

$$0.20\,\text{eV}^2 \lesssim \Delta m^2_{\text{SBL}} \lesssim 2.0\,\text{eV}^2\,, \tag{2.5}$$

[1] In the following we will denote the mass-squared differences $m_k^2 - m_j^2$ with Δm^2_{kj}.

that is allowed by the 99% CL maximum likelihood analysis of the LSND data in terms of two-neutrino oscillations [3] and by the 90% CL exclusion curves of the Bugey reactor $\bar{\nu}_e$ disappearance experiment [19] and of the BNL E776 [20] and KARMEN [21] experiments (see also [22]).[2]

The mixing in both schemes A and B is constrained by the results of neutrino oscillation experiments [10, 11, 12, 4]. In order to quantify these constraints, let us define the quantities Σ_α, with $\alpha = e, \mu, \tau, s$, as

$$\Sigma_\alpha \equiv \sum_{k=c,d} |U_{\alpha k}|^2 \,. \tag{2.6}$$

Physically Σ_α quantify the mixing of the flavor neutrino ν_α with the two massive neutrinos whose mass-squared difference is relevant for the oscillations of atmospheric neutrinos (ν_1, ν_2 in scheme A and ν_3, ν_4 in scheme B).

Let us write down the expressions, that we will need in the following, for the oscillation probabilities in short baseline experiments and in atmospheric and long-baseline experiments. The neutrino oscillation probabilities in short-baseline (SBL) experiments are given by [10]

$$P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} = \frac{1}{2}\, A_{\alpha\beta} \left(1 - \cos \frac{\Delta m^2_{\mathrm{SBL}} L}{2E} \right) , \qquad (\beta \neq \alpha) \,, \tag{2.7}$$

$$P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} = 1 - \frac{1}{2}\, B_{\alpha\alpha} \left(1 - \cos \frac{\Delta m^2_{\mathrm{SBL}} L}{2E} \right) , \tag{2.8}$$

where L is the source-detector distance, E is the neutrino energy and the oscillation amplitudes are given by

$$A_{\alpha\beta} = 4 \left| \sum_{\substack{k=a,b \\ \text{or} \\ k=c,d}} U_{\beta k} U_{\alpha k}^* \right|^2 , \qquad B_{\alpha\alpha} = 4 \Sigma_\alpha (1 - \Sigma_\alpha) \,. \tag{2.9}$$

The two expressions for $A_{\alpha\beta}$ obtained by summing over $k = a, b$ or $k = c, d$ are equivalent because of the unitarity of the mixing matrix. The oscillation probabilities of neutrinos and antineutrinos in short-baseline experiments are equal. Since the expressions (2.7) and (2.8) have the same form as the usual probabilities in the case of two-neutrino mixing, the oscillation amplitudes $A_{\alpha\beta}$ and $B_{\alpha\alpha}$ are equivalent to the usual parameter $\sin^2 2\vartheta$ used in the analysis of the experimental results (ϑ is the two-neutrino mixing angle). The average probabilities measured by short-baseline experiments that are sensitive to values of Δm^2 much smaller than the actual value of $\Delta m^2_{\mathrm{SBL}}$ are given by

$$\langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \rangle = \frac{1}{2}\, A_{\alpha\beta} \quad (\alpha \neq \beta) \,, \qquad \langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \rangle = 1 - \frac{1}{2}\, B_{\alpha\alpha} \,. \tag{2.10}$$

[2] In order to obtain reliable results, in this paper we use 99% CL limits when possible. Limits with 90% CL are used only when they are the only available information from the corresponding experiment.

The probability of $\nu_\alpha \to \nu_\beta$ transitions in atmospheric and long-baseline experiments is given by[3] [10]

$$P^{\mathrm{LBL}}_{\nu_\alpha\to\nu_\beta} = \left| \sum_{k=a,b} U_{\beta k}\, U^*_{\alpha k} \right|^2 + \left| U^*_{\alpha c} U_{\beta c} + U^*_{\alpha d} U_{\beta d} \exp\left(-i \frac{\Delta m^2_{\mathrm{atm}} L}{2E} \right) \right|^2 . \tag{2.11}$$

The average oscillation probability measured in experiments sensitive to values of Δm^2 smaller than the actual value of $\Delta m^2_{\mathrm{atm}}$ is given by

$$\langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \rangle = \left| \sum_{k=a,b} U^*_{\alpha k} U_{\beta k} \right|^2 + \left(\sum_{k=c,d} |U_{\alpha k}|^2 |U_{\beta k}|^2 \right) , \tag{2.12}$$

for both neutrinos and antineutrinos.

3. Constraints from oscillation experiments

The negative results of short-baseline $\bar{\nu}_e$ and ν_μ disappearance experiments and the results of solar neutrino experiments imply that Σ_e and Σ_μ are constrained by [10, 12]

$$\Sigma_e \leq a^0_e(\Delta m^2_{\mathrm{SBL}}) \qquad \text{and} \qquad \Sigma_\mu \leq a^0_\mu(\Delta m^2_{\mathrm{SBL}}) \quad \text{or} \quad 1 - \Sigma_\mu \leq a^0_\mu(\Delta m^2_{\mathrm{SBL}}) \,. \tag{3.1}$$

The quantities a^0_e and a^0_μ, that depend on $\Delta m^2_{\mathrm{SBL}}$, are given by

$$a^0_\alpha = \frac{1}{2} \left(1 - \sqrt{1 - B^0_{\alpha\alpha}} \right) , \qquad (\alpha = e, \mu) \,, \tag{3.2}$$

where $B^0_{\alpha\alpha}$ is the experimental upper bound, that depends on $\Delta m^2_{\mathrm{SBL}}$, for the amplitude $B_{\alpha\alpha}$, obtained from the exclusion plots of short-baseline $\overset{(-)}{\nu}_\alpha$ disappearance experiments (see [23]). From the 90% CL exclusion curves of the Bugey reactor $\bar{\nu}_e$ disappearance experiment [19] and of the CDHS [24] accelerator ν_μ disappearance experiment it follows that $a^0_e \lesssim 3 \times 10^{-2}$ for $\Delta m^2_{\mathrm{SBL}}$ in the LSND range (2.5) and $a^0_\mu \lesssim 0.2$ for $\Delta m^2_{\mathrm{SBL}} \gtrsim 0.4\,\mathrm{eV}^2$ (see [23]).

Furthermore, it is clear that the observed disappearance of atmospheric ν_μ's implies that Σ_μ, that represents the mixing of ν_μ with the two massive neutrinos responsible for atmospheric neutrino oscillations, cannot be small. Indeed, it has been shown in [12] that the up-down asymmetry of high-energy μ-like events generated by atmospheric neutrinos measured in the Super-Kamiokande experiment ($\mathcal{A}_\mu = 0.311 \pm 0.043 \pm 0.01$ [25]) implies the bound

$$1 - \Sigma_\mu \lesssim 0.55 \,. \tag{3.3}$$

Hence, Σ_μ is large also for $\Delta m^2_{\mathrm{SBL}} \lesssim 0.3\,\mathrm{eV}^2$, that is below the sensitivity of the short-baseline ν_μ disappearance experiments performed so far.

[3] When in the following we will need to write the explicit expressions for oscillation probabilities that are different for neutrinos and antineutrinos, we will consider, for simplicity, only neutrinos. The corresponding expressions for antineutrinos are obtained by taking the complex conjugate of the elements of the mixing matrix.

The constraints in eqs. (3.1) and (3.3) show that Σ_e and $1-\Sigma_\mu$ are small (in both schemes A and B). The smallness of Σ_e implies that the oscillations of ν_e's and $\bar{\nu}_e$'s in atmospheric and long-baseline experiments are suppressed. Indeed, rather strong bounds on the transition probability $1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e}$ of $\overset{(-)}{\nu}_e$'s in long-baseline experiments (LBL) have been derived in [13].

Using the fact that the oscillations of electron neutrinos are negligible in atmospheric experiments and the fact that the SUPER-KAMIOKANDE up-down asymmetry has been measured through an average over the neutrino energy spectrum and propagation distance, we can derive a lower bound for Σ_μ stronger than the one in eq. (3.3). We will use the updated [6] value of the SUPER-KAMIOKANDE up-down asymmetry,

$$\mathcal{A}_\mu \equiv \frac{D-U}{D+U} = 0.32 \pm 0.04 \pm 0.01\,, \tag{3.4}$$

where D and U are the numbers of down-going and up-going μ-like events, respectively. Summing the statistic and systematic errors in quadrature, we obtain

$$\mathcal{A}_\mu \geq 0.224 \qquad (99\%\ \mathrm{CL})\,. \tag{3.5}$$

Since the asymmetry is due to the disappearance of ν_μ's into ν_τ's or ν_s's, we have[4]

$$\mathcal{A}_\mu = \frac{\langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle - \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle}{\langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle + \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle}\,, \tag{3.6}$$

where $\langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle$ is the average survival probability of $\overset{(-)}{\nu}_\mu$'s in short-baseline experiments, that is the same as the average survival probability of downward-going atmospheric $\overset{(-)}{\nu}_\mu$'s, and $\langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle$ is the average survival probability of $\overset{(-)}{\nu}_\mu$'s in long-baseline experiments, that is the same as the average survival probability of upward-going atmospheric $\overset{(-)}{\nu}_\mu$'s. From eqs. (2.10), (2.12) and from the definition (2.6), we have

$$\langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle = 1 - 2\Sigma_\mu(1-\Sigma_\mu)\,, \tag{3.7}$$

$$\langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle = \langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle - 2|U_{\mu d}|^2(\Sigma_\mu - |U_{\mu d}|^2)\,. \tag{3.8}$$

Taking into account the obvious constraint $0 \leq |U_{\mu d}|^2 \leq \Sigma_\mu$, the long-baseline average survival probability of $\overset{(-)}{\nu}_\mu$'s is bounded by

$$\langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle - \frac{\Sigma_\mu^2}{2} \leq \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle \leq \langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle\,. \tag{3.9}$$

[4]Here we neglect corrections due to matter effects, whose calculation would require a complicated fit of the data, beyond our possibilities. However, the matter corrections are expected to be small because of the average over a wide range of neutrino energies.

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Using the lower bound in eq. (3.9), for the asymmetry (3.6) we have the upper bound

$$\mathcal{A}_\mu \leq \frac{\Sigma_\mu^2}{7\Sigma_\mu^2 - 8\Sigma_\mu + 4} \,. \tag{3.10}$$

Inverting this inequality, we get

$$\Sigma_\mu \geq \frac{4\mathcal{A}_\mu - 2\sqrt{\mathcal{A}_\mu(1-3\mathcal{A}_\mu)}}{7\mathcal{A}_\mu - 1} \,. \tag{3.11}$$

This is the lower bound for Σ_μ given by the asymmetry (3.6). Thus, in the framework of the four-neutrino schemes under consideration the asymmetry $\mathcal{A}_\mu$ cannot be larger than 1/3, a condition that is satisfied by the measured asymmetry (3.4). From eqs. (3.5) and (3.11) we obtain

$$1 - \Sigma_\mu \lesssim 0.38 \equiv a_\mu^{\rm SK} \,. \tag{3.12}$$

This is the new upper bound for $1-\Sigma_\mu$, that is more stringent than the one in eq. (3.3).

Furthermore, using expression (3.8) for $\langle P^{\rm LBL}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle$, we can derive the range of $|U_{\mu d}|^2$ that is allowed by the lower bound (3.5) for $\mathcal{A}_\mu$ (if $|U_{\mu d}|^2$ were allowed to vary in the whole range $[0, \Sigma_\mu]$, the asymmetry could vanish). After a straightforward calculation we obtain that $|U_{\mu d}|^2$ is bounded in the range

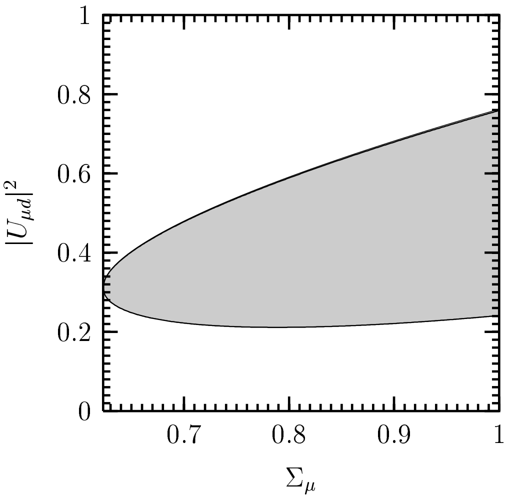

Figure 1: Allowed range for $|U_{\mu d}|^2$ as a function of Σ_μ obtained from equation (3.13) and the limit (3.5) for the Super-Kamiokande up-down asymmetry.

$$\lambda_\mu^- \leq |U_{\mu d}|^2 \leq \lambda_\mu^+ \,, \tag{3.13}$$

with

$$\lambda_\mu^\pm = \frac{\Sigma_\mu}{2} \pm \frac{1}{2}\sqrt{\frac{(1-7\mathcal{A}_\mu)\Sigma_\mu^2 - 4\mathcal{A}_\mu(1-2\Sigma_\mu)}{1+\mathcal{A}_\mu}} \,. \tag{3.14}$$

The allowed range for $|U_{\mu d}|^2$ as a function of Σ_μ obtained from the lower bound (3.5) for the asymmetry is shown by the shadowed area in figure 1. Since Σ_μ can be close to one, we have the allowed range

$$0.21 \lesssim |U_{\mu d}|^2 \lesssim 0.76 \,. \tag{3.15}$$

The same allowed range obviously applies also to $|U_{\mu c}|^2$, with the constraint (2.6) (for example, from figure 1 one can see that $|U_{\mu d}|^2 \simeq 0.21$ is possible only if $\Sigma_\mu \simeq 0.8$ and the corresponding value of $|U_{\mu c}|^2$ is 0.59, well within the range (3.15)).

Up to now we have derived the upper bounds for Σ_e and $1-\Sigma_\mu$ implied by the results of neutrino oscillation experiments. However, Σ_e and $1-\Sigma_\mu$, albeit small, cannot vanish, because non-zero values of both Σ_e and $1-\Sigma_\mu$ are required in order to generate the $\bar{\nu}_e \to \bar{\nu}_\mu$ and $\nu_e \to \nu_\mu$ transitions observed in the LSND experiment [3]. Indeed, we are going to derive lower bounds for Σ_e and $1-\Sigma_\mu$ from the results of the LSND experiment.

Let us consider the amplitude $A_{\mu e}$ of $\bar{\nu}_e \to \bar{\nu}_\mu$ and $\nu_e \to \nu_\mu$ oscillations in the LSND experiment. From eq. (2.9), using the Cauchy–Schwarz inequality, we obtain two inequalities:

$$A_{\mu e} \leq 4 \Sigma_e \Sigma_\mu \,, \tag{3.16}$$

$$A_{\mu e} \leq 4 \left(1-\Sigma_e\right)\left(1-\Sigma_\mu\right) . \tag{3.17}$$

It is clear that both Σ_e and $1-\Sigma_\mu$ must be bigger than zero and smaller than one in order to have a non-vanishing amplitude $A_{\mu e}$ in the LSND experiment.

From the inequalities (3.16) and (3.17) we obtain the upper bound

$$1-\Sigma_\mu \leq 1 - \frac{A_{\mu e}^{\mathrm{min}}}{4 a_e^0} \equiv a_\mu^{\mathrm{LSND}} \,, \tag{3.18}$$

already derived in [12], and the lower bounds

$$\Sigma_e \geq \Lambda_e^- \qquad \text{and} \qquad 1-\Sigma_\mu \geq \Lambda_\mu^- \,, \tag{3.19}$$

with

$$\Lambda_e^- = \Lambda_\mu^- = \frac{1}{2}\left(1-\sqrt{1-A_{\mu e}^{\mathrm{min}}}\right) \simeq \frac{A_{\mu e}^{\mathrm{min}}}{4} \,, \tag{3.20}$$

where $A_{\mu e}^{\mathrm{min}}$ is the minimum value of the amplitude $A_{\mu e}$ allowed by the results of the LSND experiment and the approximation is valid for small $A_{\mu e}^{\mathrm{min}}$, as follows from the LSND data. Obviously, the minimum (3.20) depends on $\Delta m^2_{\mathrm{SBL}}$ through the dependence from $\Delta m^2_{\mathrm{SBL}}$ of $A_{\mu e}^{\mathrm{min}}$.

Summarizing, from the results of all neutrino oscillation experiments we have the bounds

$$\Lambda_e^-(\Delta m^2_{\mathrm{SBL}}) \leq \Sigma_e \leq \Lambda_e^+(\Delta m^2_{\mathrm{SBL}}) \,, \qquad \Lambda_\mu^-(\Delta m^2_{\mathrm{SBL}}) \leq 1-\Sigma_\mu \leq \Lambda_\mu^+(\Delta m^2_{\mathrm{SBL}}) \,, \tag{3.21}$$

with $\Lambda_e^- = \Lambda_\mu^-$ given by eq. (3.20) and

$$\Lambda_e^+ = a_e^0 \,, \qquad \Lambda_\mu^+ = \min\left[a_\mu^0 \,,\, a_\mu^{\mathrm{SK}} \,,\, a_\mu^{\mathrm{LSND}}\right] . \tag{3.22}$$

In eq. (3.21) we have emphasized that the bounds for Σ_e and Σ_μ depend on the value of $\Delta m^2_{\mathrm{SBL}}$, that must lie in the LSND range (2.5).

The dashed line in figure 2 represents the lower bound (3.19) for Σ_e obtained from the 99% CL maximum likelihood allowed region of the LSND experiment [3]. The solid line in figure 2 is the upper bound (3.1) for Σ_e obtained from the 90% CL exclusion curve of the Bugey reactor $\bar{\nu}_e$ disappearance experiment [19].

The dash-dotted line in figure 2 shows the upper bound for Σ_e obtained from the negative result of the CHOOZ long-baseline reactor $\bar{\nu}_e$ disappearance experiment [26]. This bound has been obtained taking into account that the survival probability of $\bar{\nu}_e$'s in long-baseline experiments is bounded by [13]

$$P^{\text{LBL}}_{\bar{\nu}_e\to\bar{\nu}_e} \leq {\Sigma_e}^2 + (1-\Sigma_e)^2 \,. \qquad (3.23)$$

The CHOOZ experiment has measured a survival probability averaged over the energy spectrum [26]

$$\begin{aligned} P^{\text{CHOOZ}}_{\bar{\nu}_e\to\bar{\nu}_e} &= 1.010 \pm 0.028 \pm 0.027 \\ &= 1.010 \pm 0.039 \,, \qquad (3.24) \end{aligned}$$

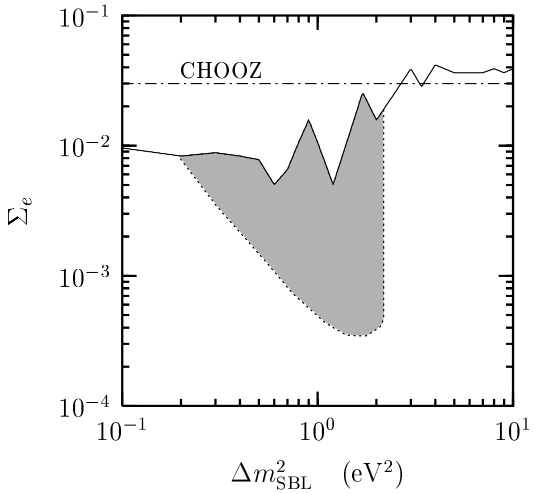

Figure 2: Allowed range for Σ_e as a function of Δm^2_{SBL} (shadowed region). The solid line is the upper bound (3.1) obtained from the exclusion curve of the Bugey experiment. The dashed line represents the lower bound (3.19) obtained from the allowed region of the LSND experiment. The dash-dotted line shows the upper bound (3.26) obtained from the negative result of the CHOOZ experiment.

where we added in quadrature the independent statistical and systematic errors. Assuming a gaussian error distribution and taking into account that the survival probability is physically limited in the interval $[0, 1]$, we obtain the 90% CL lower bound[5]

$$P^{\text{CHOOZ}}_{\bar{\nu}_e\to\bar{\nu}_e} \geq 0.942 \equiv P^{\text{CHOOZ,min}}_{\bar{\nu}_e\to\bar{\nu}_e} \,. \qquad (3.25)$$

Taking into account the fact that Σ_e is small, from eqs. (3.23) and (3.25) we obtain

$$\Sigma_e \leq \frac{1}{2}\left(1-\sqrt{1-2\left(1-P^{\text{CHOOZ,min}}_{\bar{\nu}_e\to\bar{\nu}_e}\right)}\right) = 3.0\times 10^{-2} \,. \qquad (3.26)$$

This is the upper bound represented by the dash-dotted line in figure 2. One can see that the CHOOZ bound is compatible with the Bugey bound (solid line) and a more sensitive experiment it is necessary in order to probe the range of Σ_e allowed by the results of the LSND experiment (shadowed area).

[5]The lower bound (3.25) has been obtained using a gaussian distribution normalized in the interval $[0, 1]$, according to the bayesian prescription (see [27, 16, 28]). The lower bound obtained with the Unified Approach [29] is $P^{\text{CHOOZ}}_{\bar{\nu}_e\to\bar{\nu}_e} \geq 0.945$, in reasonable agreement with (3.25).

The bounds (3.1), (3.12), (3.18) and (3.19) for $1 - \Sigma_\mu$ are illustrated in figure 3. The shadowed area enclosed by the solid line is excluded by the 90% CL exclusion curve of the CDHS [24] short-baseline ν_μ disappearance experiment using eq. (3.1) ($1 - \Sigma_\mu \leq a^0_\mu$ or $\Sigma_\mu \leq a^0_\mu$ [10, 23]). The vertically hatched area limited by the dash-dotted line is excluded by the inequality (3.12) obtained the up-down asymmetry of high-energy μ-like events generated by atmospheric neutrinos measured in the SUPER-KAMIOKANDE experiment. The diagonally hatched region is excluded by the results of the LSND experiment, taking into account also the Bugey 90% CL exclusion curve. The region above the dashed line is excluded by the upper bound (3.18) and the region below the dotted line is excluded by the lower bound (3.19). Taking into account all these constraints, the allowed region for $1 - \Sigma_\mu$ is given by the white area in figure 3.

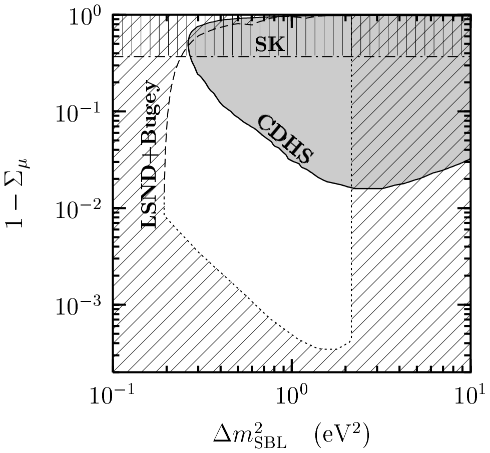

Figure 3: Allowed range for Σ_μ as a function of $\Delta m^2_{\mathrm{SBL}}$ (white region). The shadowed area enclosed by the solid line is excluded by the 90% CL exclusion curve of the CDHS experiment. The vertically hatched area limited by the dash-dotted line is excluded by the inequality (3.12) obtained from the SUPER-KAMIOKANDE up-down asymmetry. The region above the dashed line is excluded by the upper bound (3.18) and the region below the dotted line is excluded by the lower bound (3.19).

4. Long-baseline disappearance experiments

From the expression (2.11) with $\alpha = \beta$ (and the corresponding one for antineutrinos) it follows that the probability of $\overset{(-)}{\nu}_\alpha$ transitions in other states in long-baseline experiments is bounded by [13]

$$2\Sigma_\alpha(1 - \Sigma_\alpha) \leq 1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \leq 2\Sigma_\alpha \left(1 - \frac{1}{2}\,\Sigma_\alpha\right). \tag{4.1}$$

It is interesting to notice that the lower bound coincides with the average probability of ν_α transitions in other states in short-baseline experiments. Indeed, from eqs. (2.8) and (2.9) we have

$$1 - \langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \rangle = \frac{1}{2}\,B_{\alpha\alpha} = 2\Sigma_\alpha(1 - \Sigma_\alpha)\,. \tag{4.2}$$

This is obviously due to the fact that long-baseline ν_α disappearance experiments are sensitive also to short-baseline oscillations due to $\Delta m^2_{\mathrm{SBL}}$, but since these oscillations are very fast, only their average is observed in long-baseline experiments. The surprising fact is that the interference terms in the long-baseline disappearance probability are such that the disappearance in long-baseline experiments cannot be smaller than the average disappearance in short-baseline experiments. As we will see later, the analogous propriety for $\nu_\alpha \to \nu_\beta$ transitions is not true.

If we consider $\alpha = e$ in eq. (4.1), taking into account the bound (3.21) for Σ_e, we have

$$\frac{A_{\mu e}^{\min}}{2} \simeq 2\Lambda_e^-(1-\Lambda_e^-) \leq 1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e} \leq 2\Lambda_e^+ \left(1 - \frac{1}{2}\,\Lambda_e^+\right) \simeq 2a_e^0\,, \qquad (4.3)$$

with the approximations applying, respectively, for small $A_{\mu e}^{\min}$ and small a_e^0, as realized in practice. Figure 4 shows the values of the bounds (4.3) obtained from the existing data. The solid line in figure 4 shows the upper bound in (4.3) obtained from the exclusion curve of the Bugey $\bar{\nu}_e$ short-baseline disappearance experiment and the dotted line shows the lower bound in (4.3) obtained from the lower edge of the region in the $A_{\mu e}$–$\Delta m^2_{\mathrm{SBL}}$ plane allowed at 99% CL by the results of the LSND experiment. The shadowed region in figure 4 is allowed by all data and one can see that it extends below the CHOOZ bound (dash-dotted line obtained from eq. (3.25)). The lower bound for $1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha}$ lies above 7×10^{-4}, a limit that will be very difficult to reach. However, if $\Delta m^2_{\mathrm{SBL}}$ lies in the lower part of the LSND-allowed region, the lower bound $1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha}$ is substantially higher and may be reachable in the near future (see [30]).

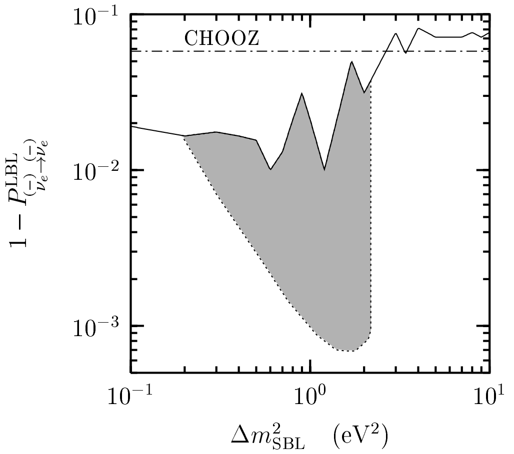

Figure 4: Allowed range for $1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e}$ as a function of $\Delta m^2_{\mathrm{SBL}}$ (shadowed region). The solid line shows the upper bound in eq. (4.3) obtained from the exclusion curve of the Bugey experiment. The dotted line shows the lower bound in eq. (4.3) obtained from the results of the LSND experiment. The dash-dotted line represents the upper bound obtained in the CHOOZ experiment.

Considering the disappearance of ν_μ's in long-baseline experiments, it is clear that, since Σ_μ is large, the corresponding probability can vary almost from zero to one. Indeed, using the limits (3.21) for Σ_μ in eq. (4.1) we obtain

$$\frac{A_{\mu e}^{\min}}{2} \simeq 2\Lambda_\mu^-(1-\Lambda_\mu^-) \leq 1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \leq \left(1 - \Lambda_\mu^-\right) \left(1 + \frac{1}{2}\,\Lambda_\mu^-\right) \simeq 1 - \frac{A_{\mu e}^{\min}}{8}\,. \qquad (4.4)$$

The expression (4.1) gives the bounds for the transition probability $1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha}$, that depends on the neutrino energy. However, if a long baseline experiment is sensitive to values of Δm^2 smaller than the actual value of $\Delta m^2_{\mathrm{atm}}$, its transition probability is given by the average (2.12). For $\alpha = \beta$ in eq. (2.12), taking into account the definition of Σ_α in eq. (2.6) and the obvious constraints $|U_{\alpha c}|^2, |U_{\alpha d}|^2 \leq \Sigma_\alpha$, one can derive the bounds

$$2\Sigma_\alpha(1-\Sigma_\alpha) \leq 1 - \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\alpha} \rangle \leq 2\Sigma_\alpha \left(1 - \frac{3}{4}\,\Sigma_\alpha\right) . \tag{4.5}$$

Therefore, only the upper bound for the average transition probability is different from the corresponding one for the transition probability, but the difference in practice is negligible for $\alpha = e$, because Σ_e is very small. Hence, the bounds in figure 4 apply also to $1 - \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_e} \rangle$.

For the average transition probability of ν_μ's into other states, a lower bound that is stronger than the lower bound implied by eq. (4.5) (that coincides with the one in eq. (4.4)) is given by the asymmetry (3.6), from which we get

$$\langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle = \frac{1-\mathcal{A}_\mu}{1+\mathcal{A}_\mu}\, \langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle . \tag{4.6}$$

Taking into account the limit (3.5) for $\mathcal{A}_\mu$ and approximating the upper bound $\langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle \leq 1 - 2\Lambda^-_\mu(1-\Lambda^-_\mu)$ with one, we obtain

$$1 - \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle \gtrsim 0.37 . \tag{4.7}$$

This is the lower bound for the average transition probability of $\overset{(-)}{\nu}_\mu$'s into other states in order to accommodate the up-down asymmetry observed in the SUPER-KAMIOKANDE experiment. We have also an upper bound for this transition probability, that is given by eq. (4.5):

$$1 - \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle \leq \begin{cases} \dfrac{\left(1-\Lambda^+_\mu\right)\left(1+3\Lambda^+_\mu\right)}{2} & \text{for} \quad \Lambda^+_\mu < \dfrac{1}{3} , \\[2ex] \dfrac{2}{3} & \text{for} \quad \Lambda^+_\mu \geq \dfrac{1}{3} , \end{cases} \tag{4.8}$$

where we have taken into account the fact that, from eqs. (3.12) and (3.22), Λ^+_μ can be as large as about 0.38. It is interesting to notice that there is the possibility that $1 - \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu} \rangle$ is larger that 1/2. Since in the case of two-neutrino mixing the average transition probability of any flavor neutrino into other states is smaller than 1/2 (equal in the case of maximal mixing), the measurement in long-baseline experiments of an average transition probability of ν_μ's into other states larger than 1/2 would be an evidence of the existence of a neutrino mass-squared difference larger than the atmospheric one and more than two-neutrino mixing. Equation (4.8) shows that this is possible in the framework of four-neutrino mixing, given the existing experimental data.

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5. Long-baseline appearance experiments

Taking into account the definition (2.9) for $A_{\alpha\beta}$, from eq. (2.11) it follows that the probability of $\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}$ transitions with $\alpha \neq \beta$ in long-baseline experiments is bounded by [13]

$$\frac{1}{4}\,A_{\alpha\beta} \leq P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}} \leq \frac{1}{4}\,A_{\alpha\beta} + \Sigma_{\alpha}\Sigma_{\beta}\,, \tag{5.1}$$

and by the unitarity upper limits

$$P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}} \leq 1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\alpha}}\,, \qquad P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}} \leq 1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_{\beta}\to\overset{(-)}{\nu}_{\beta}}\,. \tag{5.2}$$

The corresponding allowed range for $P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_{\mu}\to\overset{(-)}{\nu}_{e}}$ obtained from the experimental data has been presented in ref. [13]. The other constrained channels are $\nu_{\alpha} \to \nu_{\beta}$ with $\alpha = e, \mu$ and $\beta = \tau, s$, for which we have only the upper limits

$$P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}} \leq 1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\alpha}} \qquad (\alpha = e, \mu \quad \beta = \tau, s)\,, \tag{5.3}$$

due to the conservation of probability, with the upper limits for $1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_{e}\to\overset{(-)}{\nu}_{e}}$ and $1 - P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_{\mu}\to\overset{(-)}{\nu}_{\mu}}$ given in eqs. (4.3) and (4.4), respectively.

Confronting the interval (5.1) with the expression for the average probability of $\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}$ transitions in short-baseline experiments given in eq. (2.10), one can see that, as anticipated in the discussion after eq. (4.2), the probability of long-baseline $\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}$ transitions can be smaller than the average probability of short-baseline $\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}$ transitions. This is due to interference effects produced by $\Delta m^2_{\mathrm{atm}}$. Hence we have

$$P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}} < \langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}} \rangle \qquad \text{(possible)}\,. \tag{5.4}$$

For a fixed source-detector distance in a long-baseline experiment, the realization of the possible inequality (5.4) depends on the value of the neutrino energy. Therefore, using a detector with sufficient energy resolution (or with a sufficiently narrow-band beam and varying the neutrino energy) it is possible to check experimentally if the inequality (5.4) can be realized. The inequality (5.4) is interesting because it can be realized only if there are at least two mass-squared differences, $\Delta m^2_{\mathrm{SBL}}$ and $\Delta m^2_{\mathrm{atm}}$. Indeed, it is impossible in the case of two-neutrino mixing that implies, with an obvious notation,

$$P^{\mathrm{LBL},2}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}} = \langle P^{\mathrm{SBL},2}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}} \rangle \qquad \text{only } \Delta m^2_{\mathrm{SBL}}\,, \tag{5.5}$$

$$P^{\mathrm{LBL},2}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}} \geq \langle P^{\mathrm{SBL},2}_{\overset{(-)}{\nu}_{\alpha}\to\overset{(-)}{\nu}_{\beta}} \rangle = 0 \qquad \text{only } \Delta m^2_{\mathrm{atm}}\,. \tag{5.6}$$

However, the possibility of the inequality (5.4) is not unique to the four-neutrino schemes, and it can be realized, for example, in three-neutrino schemes with $\Delta m^2_{31} = \Delta m^2_{\mathrm{SBL}}$ and $\Delta m^2_{21} = \Delta m^2_{\mathrm{atm}}$.

The experimental check of the inequality (5.4) could provide some information on the relative phases of the elements of the mixing matrix. Indeed, confronting eqs. (2.11) and (2.10), taking into account of the definition (2.9) for $A_{\alpha\beta}$, one can see that the inequality (5.4) for neutrinos is realized if

$$\mathrm{Re}\left[U_{\alpha c}U_{\beta c}^{*}U_{\alpha d}^{*}U_{\beta d}\exp\left(-i\frac{\Delta m_{\mathrm{atm}}^{2}L}{2E}\right)\right] < \mathrm{Re}\left[U_{\alpha c}U_{\beta c}^{*}U_{\alpha d}^{*}U_{\beta d}\right]. \tag{5.7}$$

The product $U_{\alpha c}U_{\beta c}^{*}U_{\alpha d}^{*}U_{\beta d}$ is invariant under rephasing of the mixing matrix, i.e. the transformations $U_{\alpha k} \to e^{i\gamma_\alpha}U_{\alpha k}$ and the transformations $U_{\alpha k} \to e^{i\gamma_k}U_{\alpha k}$. Using these transformations we can consider, without loss of generality, $U_{\alpha c} = |U_{\alpha c}|$, $U_{\beta c} = |U_{\beta c}|$, $U_{\alpha d} = |U_{\alpha d}|$, $U_{\beta d} = |U_{\beta d}|e^{i\xi}$ and write the inequality (5.7) as

$$\cos(\xi - \varphi) < \cos\xi\,, \tag{5.8}$$

where $\varphi \equiv \Delta m_{\mathrm{atm}}^{2}L/2E$. If we confine ξ in the interval $[0, 2\pi)$, the inequality (5.8) is not realized for any value of φ only if

$$\xi = \pi\,, \tag{5.9}$$

and it is realized for all values of $\varphi \neq 0$ only if

$$\xi = 0\,. \tag{5.10}$$

Since the two extreme cases (5.9) and (5.10) correspond to the absence of CP-violating phases in the α, β-$1, 2$ sector of the mixing matrix, it is clear that the observation of the inequality (5.4) for some, but not all values of φ would be an evidence in favor of CP violation in the lepton sector due to neutrino mixing. This fact can also be seen by noting that if CP is conserved all the elements of the mixing matrix are real and there are two possibilities:

$$\begin{aligned}
&1)\ \ \frac{1}{4}A_{\alpha\beta} = |U_{\alpha c}U_{\beta c} + U_{\alpha d}U_{\beta d}|^2 = (|U_{\alpha c}U_{\beta c}| + |U_{\alpha d}U_{\beta d}|)^2 \quad \text{if} \quad U_{\alpha c}U_{\beta c}U_{\alpha d}U_{\beta d} > 0\,,\\
&2)\ \ \frac{1}{4}A_{\alpha\beta} = |U_{\alpha c}U_{\beta c} + U_{\alpha d}U_{\beta d}|^2 = (|U_{\alpha c}U_{\beta c}| - |U_{\alpha d}U_{\beta d}|)^2 \quad \text{if} \quad U_{\alpha c}U_{\beta c}U_{\alpha d}U_{\beta d} < 0\,.
\end{aligned} \tag{5.11}$$

In the first case we have

$$1) \qquad \left|U_{\alpha c}U_{\beta c} + U_{\alpha d}U_{\beta d}e^{-i\varphi}\right|^2 \leq \frac{1}{4}A_{\alpha\beta}\,, \tag{5.12}$$

and

$$1) \qquad P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \leq \langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} \rangle \qquad \text{always}\,. \tag{5.13}$$

In the second case we have

$$2) \qquad \left|U_{\alpha c}U_{\beta c} + U_{\alpha d}U_{\beta d}e^{-i\varphi}\right|^2 \geq \frac{1}{4}\,A_{\alpha\beta}\,, \tag{5.14}$$

and

$$2) \qquad P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\alpha\to\overset{(-)}{\nu}_\beta} \geq \langle P^{\mathrm{SBL}}_{\overset{(-)}{\nu}_\alpha\to\overset{(-)}{\nu}_\beta}\rangle \qquad \text{always}\,. \tag{5.15}$$

The inequality (5.15) has the same origin as the inequality (5.6) in the case of two-neutrino mixing.

If $\Delta m^2_{\mathrm{atm}}$ is known and the the detector has a sufficient energy resolution (or with a sufficiently narrow-band beam and varying the neutrino energy), it is even possible to measure the CP-violating phase ξ by looking at the variation of the transition probability as a function of φ. For example, we have

$$P^{\mathrm{LBL}}_{\nu_\alpha\to\nu_\beta}(\varphi=\pi) - P^{\mathrm{LBL}}_{\nu_\alpha\to\nu_\beta}(\varphi=0) = -4|U_{\alpha c}U_{\beta c}U_{\alpha d}U_{\beta d}|\cos\xi\,, \tag{5.16}$$

$$P^{\mathrm{LBL}}_{\nu_\alpha\to\nu_\beta}\left(\varphi=\frac{3}{2}\pi\right) - P^{\mathrm{LBL}}_{\nu_\alpha\to\nu_\beta}\left(\varphi=\frac{1}{2}\pi\right) = -4|U_{\alpha c}U_{\beta c}U_{\alpha d}U_{\beta d}|\sin\xi\,. \tag{5.17}$$

The ratio of eqs. (5.17) and (5.16) gives $\tan\xi$ independently from the value of $|U_{\alpha c}U_{\beta c}U_{\alpha d}U_{\beta d}|$.

Let us finally consider the average transition probabilities that are measured in long-baseline experiments that are sensitive to values of Δm^2 much smaller than the actual value of $\Delta m^2_{\mathrm{atm}}$. Considering $\overset{(-)}{\nu}_\mu\to\overset{(-)}{\nu}_e$ transitions, from eq. (2.12) we have

$$\langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu\to\overset{(-)}{\nu}_e}\rangle = \frac{1}{4}\,A_{\mu e} + |U_{\mu c}|^2|U_{ec}|^2 + |U_{\mu d}|^2|U_{ed}|^2\,. \tag{5.18}$$

Taking into account the bounds (3.21) for Σ_e and Σ_μ, the allowed range (3.13) for $|U_{\mu d}|^2 = \Sigma_\mu - |U_{\mu c}|^2$ and the constraint $0 \leq |U_{ed}|^2 \leq \Sigma_e$ for $|U_{ed}|^2 = \Sigma_e - |U_{ec}|^2$, one can find that the average transition probability (5.18) is bounded by

$$\frac{1}{4}\,A_{\mu e} + \lambda^-_\mu(\mathrm{min})\Lambda^-_e \leq \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu\to\overset{(-)}{\nu}_e}\rangle \leq \frac{1}{4}\,A_{\mu e} + \lambda^+_\mu(\mathrm{max})\Lambda^+_e\,. \tag{5.19}$$

The lower bound is reached for $|U_{ed}|^2 = 0$ and $|U_{\mu d}|^2 = \Sigma_\mu - \lambda^-_\mu(\mathrm{min})$ or $|U_{ed}|^2 = \Sigma_e$ and $|U_{\mu d}|^2 = \lambda^-_\mu(\mathrm{min})$. The quantity $\lambda^-_\mu(\mathrm{min})$ is the minimum value of λ^-_μ in the allowed range for Σ_μ. For the values of $\Delta m^2_{\mathrm{SBL}}$ in which Σ_μ is limited by the Super-Kamiokande asymmetry constraint (3.12), the minimum of λ^-_μ, shown in figure 1, occurs for

$$\Sigma_\mu = \frac{8\mathcal{A}_\mu - \sqrt{2(1-3\mathcal{A}_\mu)(1+\mathcal{A}_\mu)}}{2(7\mathcal{A}_\mu - 1)}\,. \tag{5.20}$$

The upper bound in eq. (5.19) is reached for $|U_{ed}|^2 = 0$ and $|U_{\mu d}|^2 = \Sigma_\mu - \lambda^+_\mu(\mathrm{max})$ or $|U_{ed}|^2 = \Sigma_e$ and $|U_{\mu d}|^2 = \lambda^+_\mu(\mathrm{max})$. The quantity $\lambda^+_\mu(\mathrm{max})$ is the maximum value of λ^+_μ in the allowed range for Σ_μ, that, as shown in figure 1, occurs always for the maximum allowed value of Σ_μ, $\Sigma^{\max}_\mu = 1 - \Lambda^-_\mu$.

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Figure 5 shows the allowed range (5.19) as a function of $\Delta m^2_{\mathrm{SBL}}$ (shadowed region). The dashed line in figure 5 represents the unitarity upper bound $\langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu\to\overset{(-)}{\nu}_e}\rangle \leq 1 - \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_e\to\overset{(-)}{\nu}_e}\rangle$, with the upper bound for $1 - \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_e\to\overset{(-)}{\nu}_e}\rangle$ given by eq. (4.5). Since $\Lambda^+_e = a^0_e$ is small, the dashed line in figure 5 practically coincides with the solid line in figure 4.

The other constrained channels are $\nu_\alpha \to \nu_\beta$ with $\alpha = e, \mu$ and $\beta = \tau, s$, for which $\langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\alpha\to\overset{(-)}{\nu}_\beta}\rangle$ has only the unitarity upper limits given by the average of eq. (5.3).

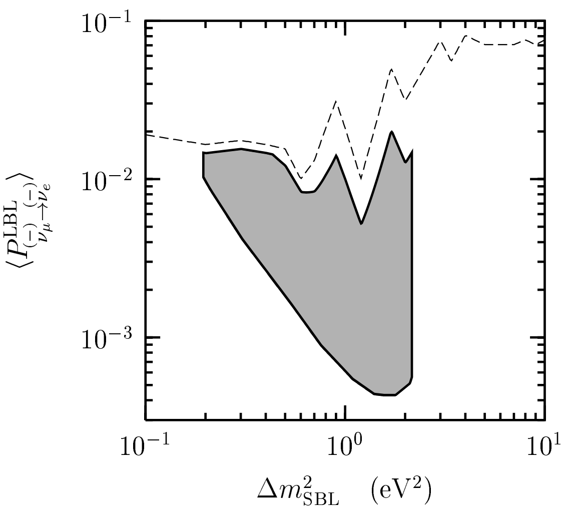

Figure 5: Allowed range (5.19) for $\langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu\to\overset{(-)}{\nu}_e}\rangle$ as a function of $\Delta m^2_{\mathrm{SBL}}$ (shadowed region). The dashed curve represents the unitarity constraint $\langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_\mu\to\overset{(-)}{\nu}_e}\rangle \leq 1 - \langle P^{\mathrm{LBL}}_{\overset{(-)}{\nu}_e\to\overset{(-)}{\nu}_e}\rangle$.

6. Conclusions

We have presented the constraints on the neutrino mixing parameters obtained from the results of neutrino oscillation experiments in the framework of the two four-neutrino schemes (2.1) that are compatible with all neutrino oscillation data. We have shown that the parameters Σ_e and $1 - \Sigma_\mu$ defined in eq. (2.6) are small, but do not vanish. Their allowed ranges are presented in figures 2 and 3. We have also derived the bounds for the mixing of ν_μ with the two massive neutrinos whose mass-squared difference is responsible for atmospheric neutrino oscillations (eq. (3.13)). In sections 4 and 5 we presented the bounds for the oscillation probabilities in long-baseline experiments (some of which have been derived already in ref. [13]). We have also shown that a comparison of the probability measured in long-baseline appearance experiments with the corresponding average probability measured in short-baseline experiments may give information on the relative phases of the relevant elements of the mixing matrix and, in particular, may provide an evidence for the existence of CP violation in the lepton sector. This method may be more convenient than the traditional method based on the comparison of the oscillation probabilities of neutrinos and antineutrinos if only a neutrino or antineutrino beam is available.

In this paper we have considered long-baseline experiments neglecting matter effects, that are rather complicated in the framework of four-neutrino mixing and will be discussed in detail in another article [18]. We think that it is important to discuss long-baseline oscillations in vacuum for at least two reasons. First, because vacuum oscillations can be treated analytically and all the effects can be understood in a clear way. The matter effects, that often must be calculated numerically, can be appreciated as corrections (sometimes large) to the oscillations in vacuum. Second, in order to measure accurately the properties of neutrinos without the uncertainty and complications caused by matter effects, it may be possible, in some indeterminate future, to perform a long-baseline experiment in vacuum. For example, it may be possible to produce a neutrino beam in a orbital station (existing space stations orbit at an altitude between 300 and 400 km) and shoot it to some detector(s) on the Earth. This experiment would allow to measure accurately the properties of neutrinos when the beam does not cross the Earth and, with this knowledge, to perform an accurate tomography of the Earth using the matter effects when the beam crosses the Earth. At present such an experiment is fanciful, but we know that many experiments have been performed up to now that were unthinkable or believed to be impossible in the past.

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